## Introduction

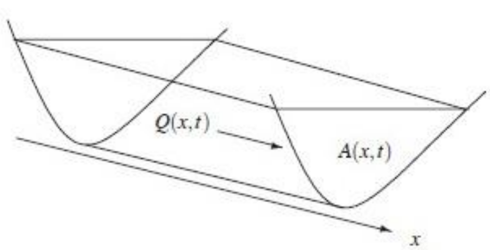
Considerable work has been devoted to the numerical solution of the {Shallow-Water Equations} (SWE) not only for their inherent importance as regards modeling of many physical processes, ranging from river and channels flows to estuarine circulation and floods due to the dam or dike failure, but also, for their mathematical difficulties, namely its non-linearity can give rise to discontinuous solutions currently referred to as bores or jumps. In fact, the SWE constitutes a nonlinear system of partial differential equations (in one and two dimensions) of the hyperbolic type with a nonlinear source term.

Many hydraulic situations can be described by means of one-dimensional model, either because a more detailed resolution is unnecessary or because the flow is markedly one-dimensional. The fundamental hypothesis implied in the numerical modeling of river flows are formalized in the equations of unsteady open channel flow. They can be derived, from instance, from mass and momentum control volume analysis and are a simplified model of a very complex phenomenon but they are considered an adequate description for most of the problems associated with open channel and river flow modeling under the St Venant hypotheses.

## Mathematical Model

### *Conservative Form of One-Dimensional St-Venant Equations*

One-dimensional open-channel flow is usually described in terms of water depth and discharge, and the evolution of these quantities is taken to be governed by the Saint-Venant equations, which simply express the conservation of mass and momentum along the flow direction.




The 1D unsteady shallow water flow can be written in the form St-Venant system can be written in different form, we consider the conservative form of the equation.

$$\partial_t \boldsymbol{U} + \partial_x \boldsymbol{F}(\boldsymbol{U}) = \boldsymbol{G} \qquad (1)$$

where

$U = \begin{pmatrix} A \\ Q \end{pmatrix}$

$F = \begin{pmatrix} Q \\ Q^2/A + gI_1 \end{pmatrix}$

$G = \begin{pmatrix} 0 \\ gI_2 + gA(S_f - S_b) \end{pmatrix}$

with F (set of fluxes of each conserved quantities) flux tensor and G source terms. Other variables are the following

- **U**: state vector of the state variables, respectively wetted area and discharge of section flow
- **F** : physical flux, convective and pressure
- $\boldsymbol{S_f}$: friction term expressed by the Manning formula
- $S_b$: bottom slope term expressed as derivative of the bathymetry
- **A**: wetted cross-section area which depends **(x,h(x,t))**
- **g**: acceleration of gravity
- $\boldsymbol{I_1}$: pressure term given by $\int_0^{h(x)} (h(x) - \vartheta)\sigma(x,\vartheta)\, d\vartheta$
- $\boldsymbol{I_2}$: and given by $\int_0^{h(x)} (h(x) - \vartheta) \left[\frac{\partial \sigma(x,\vartheta)}{\partial x}\right] d\vartheta$ section width variation along x

with 'h' water depth, $\sigma$ width for a fixed depth and $\vartheta$ depth integration variable along y axis.

This form of the equations emphasizes the conservative character of the system in the absence of source terms. This equation expresses simply that the quantity U inside a volume depends only on the flux at the surface (no source inside the volume).

The equation system (1) can be approximated as a system of ordinary differential equation in t:

$$U_t = L_\Delta(U;t) \qquad (2)$$

where $U_t$ is the time derivative of the solution **U** and the operator $L_\Delta$ is the discrete approximation to the continuous convection and source operator of equation (1):

$$L_\Delta(U) \approx \left(-\frac{d\,f(u)}{dx} + s(u)\right)_{x\_i}$$

Writing equation in the above form allows us to implement separately the discretization of the differential operator $L_\Delta$ and the time scheme used for the evolution of the solution. This is called Method-of-lines (sometime semi-discrete).

Next the definition of $L_\Delta(U)$ for various spatial nodes are defined.

where $L_\Delta$ represents the differential operator in space i.e., one which does contain spatial derivative

$$L_\Delta = \partial_x F - \Delta t\, S - \Delta t\, P \qquad (3)$$

## Numerical Method

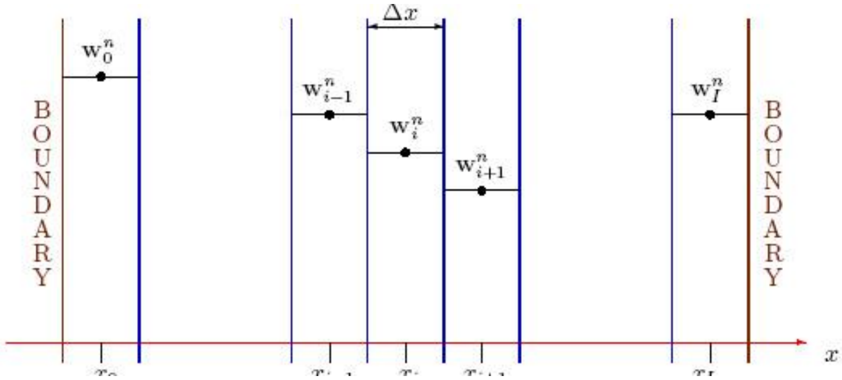
### *Finite Volume Discretization*

The main step is the choice of the discretization method of the mathematical formulation and involves two components, the *space discretization* and the *equaion discretization*. The space discretization consists of setting up a mesh or a grid by which the continuum of space is replaced by a finite number of points where the numerical values of the variables will have to be determined. The error of the numerical simulation has to tend to zero when the mesh size tends to zero, and the rapidity of this variation will be characterized by the *order* of the numerical discretization of the equations.

### *Grid*

A uniform Cartesian grid is used to discretize the domain. In the 1-D case the domain $x \in [xmin, xmax]$ of the equation is discretized as a series of points $\mathrm{x\_i}, \mathrm{i} = 0, \ldots, \mathrm{N} + 1$. Here 'i' is the spatial node number corresponding to the location $x_i = x\_\min + i\ dx$, where 'dx' is the grid spacing. The domain of the problem is represented by a collection of simple domains, called cells.

The problem consists of evaluating a discrete equation on each cell, the physical process is approximated by functions of desired type (polynomials or otherwise), and an algebraic equation relating physical quantities at selective points, called nodes, of the element are developed.




### *Spatial Discretization (Explicit Finite Difference Scheme)*

Once the mesh has been defined the equations can be discretized, leading to the transformation of the differential or integral equations to discrete algebraic operations involving the values of the unknowns at the mesh points. The basis of all numerical methods consists of this transformation of the physical equations into an algebraic, linear or non-linear, systems of equations.

Discrete representation of the numerical scheme that we used to solve the One-Dimensional St-Venant equations. A conservative finite difference method is used to solve these equations. The differential system is integrated over each control volume to produce the discrete equivalent of the conservation law

$$U_i^{n+1} = U_i^n - \lambda \left\{\mathrm{F}_{\mathrm{i}+1/2}^{\mathrm{n}} - \mathrm{F}_{\mathrm{i}-1/2}^{\mathrm{n}}\right\} - \Delta\mathrm{t}\, \mathrm{S}_{\mathrm{i}} - \Delta t\, P_i \quad (4)$$

where $F_{i\pm1/2}$ is the numerical flux of the state variables U and is expressed as follow

$$F_{i+1/2} = F\left(U\left(x_{i+1/2}, t\right)\right).$$

and F the physical flux of the state's variables and S represent source terms (friction and bottom). Friction terms are evaluated according to the Manning formula. We now have an equation which expresses the time evolution of a mean cell value in terms of the flux.

This integration technique forms the basis of what is known as the finite volume method. The specific difference between various finite volume schemes is the way in which they approximate the interface convective flux $F_{i\pm 1/2}$. This method can reproduce the discontinuities of a flow regime variations without incurring instabilities of the solution.

## Godunov-Type Scheme

The Godunov-type approach use either exact or approximate Riemann solutions between two adjacent cell to calculate the flux $\mathrm{F}_{i+1/2}^{n}$through the interface between them. The Riemann problem is a particular initial value problem (IVP), which consist of a conservation law or a system of conservation laws with a discontinuous initial solution.

## Temporal Discretization

With the discrete operator L now defined, equation (2) could be solved by a wide variety of standard numerical techniques, which have been developed for years for large systems of ordinary differential equations. Here, an explicit two-stage of second order is chosen. Given the solution $U_i$at previous time step $t_n$, the solution at $U_i$at next time step $t_{n+1}$ is constructed.

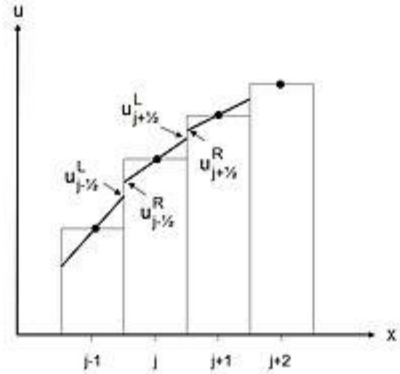
$$Ut_i = Un_i$$

$$Ut_i = Un_i + dt \sum_{k=0}^{n} a_{jk} * L_\Delta(Ut; t) \quad (5)$$

where $Ut_i$ are the intermediate solution states.

## MUSCL Reconstruction

Below the graph show the process of reconstruction of the state variables at the interface.



## *Slope Limiter*

Since we are modeling mathematical functions that have strong gradient (peak function), we need to use some technique to limit the slope of the function (called a slope limiter) thus to avoid spurious oscillations. There are many slopes limiter function, one of the most popular is the "minmod" function. It takes the minimum of adjacent gradient (of each cell)