A test field calibration to validate shallow-water codes: the Case of Sainte-Marguerite River

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Technical Report CERCA R00-6

Date: May 6, 2000

Abstract

This paper reports on a rigourous field testing for shallow water flows over complex bathymetries. A finite element code, "aquadyn"[1] is used to solve numerically the steady-state Saint-Venant equations in two-dimensions. The software has been calibrated on a 2km section of the Sainte-Marguerite river for which the water level and discharge at several points along the map was available. In this case the data field has been used only for comparison with the output of the numerical simulation. The case of study is a very strong validation test because all the difficulties are present: torrential high speed regime, transition torrential-fluvial, variable section, complex bathymetry, presence of an island. The only critical parameter that requires to be carefully adjusted by field experienced engineers is the Manning. By specifying adequate boundary conditions at the upstream end of the chanel and the water level downstream the results are in good agreement with the data with an error of less than a few percent locally. The output of our simulation could be used as a benchmark to validate shallow-water codes.

1 Introduction

Mathematical simulation of hydraulic phenomena is becoming increasingly important in engineering practice since it offers the possibility of cheaply evaluating the response of hydraulic systems to a variety of practical situations (Cunge 1991). Among these, rapidly varied open channel flow possesse certain features that make it important to predict but most difficult to compute. Supercritical torrential flows with hydraulic jumps and bores are ackward to represent even when a shock-capturing method (Hirsh 1988) is employed.

For applications in civil engineering, there are several complicating factors: turbulence, geometrical description, a lack of data to define parameters, scale size. Specially, the interaction between the main channel flow and the floodplain flow requires a full 2D model that should also be able to simulate the drying and wetting processes in the floodplain, in the main channel (the emergence of submergence of an island) and in the wetland. Ideally, a good numerical model for river-bassin simulation should be able to handle simulateously the following features:

- complex topography;
- subcritical and supercritical flow;
- steady and unsteady flow;
- smooth flow and discontinuous flow;
- wetting or drying of floodplain;

The software we test in this study is "aquadyn" from "HydroSoft" used for the simulation of shallow water flows in rivers, lakes or estuaries. The code uses the Finite element model with a Galerkin method and can be used for steady or unsteady-flow simulations. Flow regimes can be subcritical or supercritical as well as gradually varied discontinuous flow. The wetting or drying of portions of the floodplain in a river-basin can be handled. Hydraulic structures and internal boundary can be specified. Non standard boundary conditions such as discharge inflow points, water stages, and discharge-stage can also be specified (Aquadyn 1995).

A test calibration case is a portion of the Ste-Marguerite river in Quebec. The model results have been compared with the data from a physical-model-study conducted by *Groupe Lasalle* in Montreal and also with field data from the Ste-Marguerite river collected by *Hydro-Québec* environmental depart-

[1] from Hydrosoft Energie Inc.

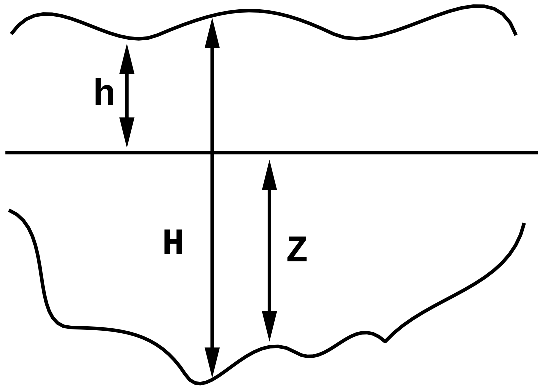

Figure 1: The relation between H = water level, h = water depth and Z = bed elevation or bathymetry

ment.

2 Governing Equations

The Saint-Venant equations (Maidment 1993, Kundu 1990) describe the motion of shallow water with a free surface and are obtained by integration of the three-dimensional Navier-Stokes equations over the depth, with the assumption of hydrostatic vertical pressure distribution, i.e. negligible vertical accelerations. This is certainly the case when most of the dynamics is captured by neglecting vertical motion. We have to solve:

$$\frac{\partial h}{\partial t} + \frac{\partial (h\,U)}{\partial x} + \frac{\partial (h\,V)}{\partial y} = 0 \qquad (1)$$

$$\frac{\partial U}{\partial t} + U\,\frac{\partial U}{\partial x} + V\,\frac{\partial U}{\partial y} + g\,\frac{\partial H}{\partial x} + g\,U\frac{\sqrt{U^2+V^2}}{C^2 h} - \frac{\partial}{\partial x}(\nu_t \frac{\partial U}{\partial x}) - \frac{\partial}{\partial y}(\nu_t \frac{\partial U}{\partial y}) = 0 \qquad (2)$$

$$\frac{\partial V}{\partial t} + U\,\frac{\partial V}{\partial x} + V\,\frac{\partial V}{\partial y} + g\,\frac{\partial H}{\partial y} + g\,V\frac{\sqrt{U^2+V^2}}{C^2 h} - \frac{\partial}{\partial x}(\nu_t \frac{\partial V}{\partial x}) - \frac{\partial}{\partial y}(\nu_t \frac{\partial V}{\partial y}) = 0 \qquad (3)$$

where the variables are defined as follows:

H	= water level
h	= water depth ($h = H - Z$) (Fig. 1)
Z	= bed elevation or bathymetry
U, V	= velocity horizontal components
C	= Chézy coefficient
ν_t	= turbulent viscosity

Here the full nonlinear Shallow-water equations are needed to account for the torrential regime. The Chézy term (Dingman 1984) accounts for the lost of energy by friction mechanism at the bed level:

$$C = \frac{1}{n} h^{1/6} \qquad (4)$$

where: n is the Manning coefficient ($M^{-1/3}.s$) which depends on the nature of the bed surface.

The turbulent (eddy) viscous term (Dingman 1984) is used to model the isotropic 2D momentum dissipation by turbulence due to vortices smaller than the space resolutioni ($\approx 20m$). The eddy viscosity ν_t is:

$$\nu_t = \gamma\,A\sqrt{2\frac{\partial U}{\partial x} + 2\frac{\partial V}{\partial y} + (\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x})^2} \qquad (5)$$

where: A is the area of an element (here a triangle) and γ is an algebraic coefficient to be adjusted in terms of the observed energy loss due to channel contraction or expansion or due to variations of bed elevation for instance.

Since Reynolds numbers are of the order of the million in river flows, the kinematic viscosity is totally negligible in hydrological cases.

The boundary conditions are "no-slip" at the banks of the river and absorbing downstream. Water levels as well as discharges can be specified both in upstream and downstream of the river section to be modelled.

3 Numerical Simulation

The need for efficient and accurate numerical schemes to solve these equations is at present of primary concern to computational hydraulics. The model used in *aquadyn* is a time dependent Galerkin finite element scheme (Reddy, 1993) that has been used here in a steady state mode.

The river-basin network is represented by an unstructured grid system using six nodes triangular el-

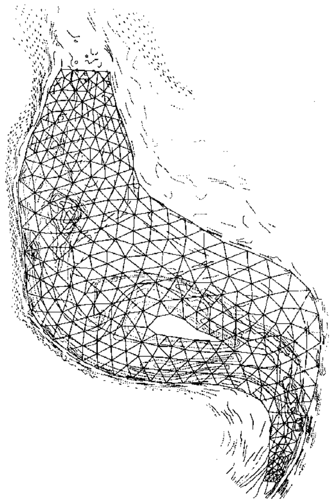

Figure 2: The network used for the Sainte-Marguerite river section to model.

ements (Fig. 2) (Dhatt and Touzot, 1981, Pinder and Gray 1977) which are located to appropriately represent the flow regime. This element has satisfied all the tests of stability and precision for transient and steady state real life flows. The choice of an adapted mesh requires experience and preliminary knowledge of the flow field.

The simulation proceeds as follows:

3.1 Initial and Boundary Condition

To start a numerical simulation we must specify the initial velocity values U, V and water level h for all nodes (i, j). In the case of a steady flow, the initial state is used as the first approximate solution of the iterative solver. Good initialization may significantly reduce the time solver required to converge to the solution at the desired accuracy. Sometimes, convergence can only be achieved if the initial is well chosen.

Proper initial conditions are obtained by running the model with uniform steady state flow. Once the model is stabilized (physical characteristic of the flow water profile and velocity profile and line of energy), the real simulation may triggered, the resulting flow state of the pre-simulation being used as the initial condition. The number of iteration for the solution is 30, an evident power of the implicit Newton method.

A consistent set of boundary conditions is required to complement the flow equations. The local value of the Froude number is what determines the flow regime and, accordingly, the correct number of boundary conditions to be applied.

In the particular case of solid walls limiting the flow fields, the velocity is projected into the tangential and normal directions to the wall. Then the normal component V_n is set equal to zero in order to represent no advective flux through the solid boundary. This is the case at the boundaries of an island or at the river banks.

At upstream and downstream boundaries, we ban specify the discharge Q or the water level H.

Finally, at the bottom and for each node (i, j), we have to specify the Manning coefficient $N(i, j)$ (Dingman 1984). It is not constant over the whole domain and is responsible for energy losses and the change of regime at the surface. The turbulent viscosity coefficient $\nu(i, j)$ is also a parameter to be adjusted and but is set constant over wet elements.

3.2 Calibration and Validation of the Model

A fundamental aspect when dealing with numerical schemes is to be able to check their predictions against suitable test problems, preferably ones for which an exact solution is available. This is not the case for the example we present in this technical report. The procedure to follow is then first to make a *calibration* on a well documented regime and secondly to ensure the *validation* on another regime.

We proceed as follows:

1: We build the mesh. This is a very delicate operation. The mesh should follow the direction of the stream. The main reason is that there are strong grid-viscosity errors with the finite element method and this artificial dissipation is anisotropic. In addition, meshing of the domain was constrained by the necessity to retain a sufficient density of elements in the main channel area to adequately represent the channel cross section.

2: We interpolate the bathymetry on the mesh points. There are various ways of doing this. Simple gridding schemes routinely used in converting irregularly gridded images to regular grids like for instance bilinear interpolation on Delaunay triangulations do not work well. On the other hand, kriging via semi-variogram gives informations about the spatial variability of the data and the interpolation distance is shorter than the correlation distance (Tranchant and

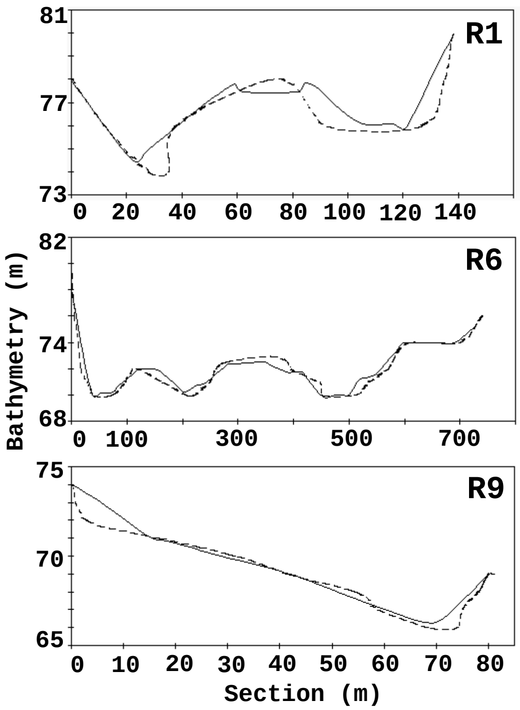

Figure 3: Interpolation (dotted lines) of the bathymetry (full line) sections (at points R1, R6 and R9) using a kriging technique gives satisfactory results exept at the sides when the profile is too stiff. The section numbers correspond to Fig. 5.

Vincent 2000).

The interpolated bathymetry is then manually compared with the actual one and further interpolations are made until the desired accuracy has been obtained. This is shown in Fig. 3. There is a good qualitative agreement between the real topology and the interpolated. The regions where there are steep gradient like for instance the side of the river are more difficult to represent.

3: The building of the initial condition requires the use of the energy slope, the Froude number, the Reynolds number, the mean stream velocity.

4: We are now ready to calibrate the model. For that we fix the turbulent viscosity and Manning parameters. At the moment, this is done by hand. We also have to fix the downstream and upstream values

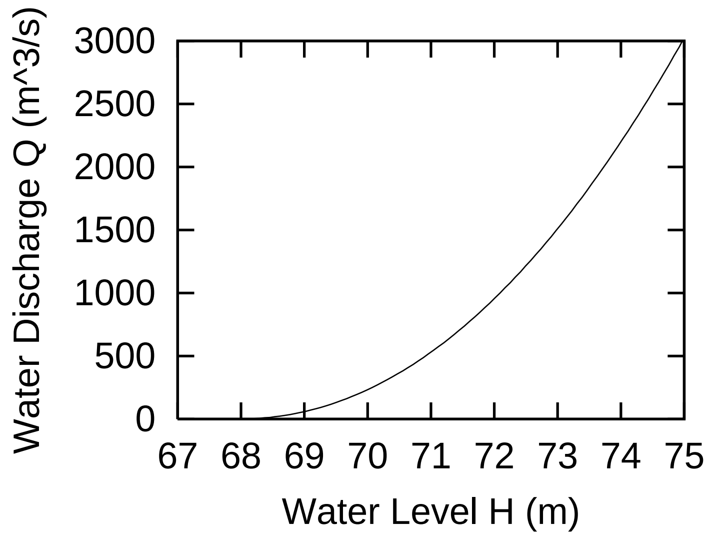

Figure 4: "Relation niveau-débit" curve supplied by Hydro-Québec. "SM 3 - Conditions naturelles. Relation Niveau-Débit - Point R9 (356.000E, 5818.189 N)"

(see above section) and the minimum water depth.

At upstream boundary, we have choosen to specify the discharge ($Q = 2500 m^3 s^{-1}$) and at downstream boundary we have fixed the water level ($H = 74.m$).

The error is finally estimated by computing the difference between the water levels at points $R1$ to $R9$ on the measured data and on the calibration run (Table 1). Only point $R9$ is both a grid point and a point of direct measurement. The result is quite remarkable. At points R4, R5, R6, R7 and R8, the result is not as good because we had to interpolate the bathymetry. It could be improved by further adjusting the Manning parameter in these areas. However it is of the order of the percent and the best possible result in this case. Taking into account that the mean depth is about $h = 7.5m$ and that errors less than 15% are usually regarded as acceptable, we think that our results could be used as a benchmark by the hydrological community.

The Manning and turbulent parameters are then reajusted manually until the error is minimal. This process may be done automatically by an optimal control technique (Poeter and Hill 1997, Maidment 1993, Louis 1989).

- The second step is the *validation*. To validate, we have to reproduce another regime of the river. We intend to use the measured variation of the water level $H(m)$ versus the observed discharge at point

"R9" [2] (see Fig. 4). This step is particularly important since it confirms that the empirical constants used for calibration are correct. This step has not been done in this study and is postponed to a future work.

3.3 Internal Boundaries: the wet-and-dry technique

During the iterative process of reaching a steady state, the water depth may become too small at certain portion of the river basin (i.e., subarea or flow element). One could remesh the grid but this would be too much time consuming. In this case, we artificially maintain a minimum water layer in order to avoid infinity in the manning friction term. However this slows down the convergence process because the stiffness matrix elements are now modified by the addition of h_{min}. Moreover, the conservation of mass is no more verified and this may lead to a wrong discharge.

An alternative would be to use the fictitious domain technique but it is still the object of research (Glowinsky *et al.* 1993 1994, Matthews *et al.* 1996).

4 Results: The Sainte-Marguerite River Section

The aquadyn model was used to simulate the flow over a 2 km Ste-Marguerite river section called " Gallerie du canal de fuite " (Hydro-Québec: project SM3). The river consists of a channel which can be divided in three parts. An upper part of about $200m$ width. A mean part of $450m$ width and finally a narrower section of $50m$ width. This river was selected because of the complexity of the flood plain topography which includes flood plain embankments, a channel bifurcation and a island (Fig. 5).

In the upstream flow of the river the flow is torrential with an hydraulic jump. The central part of the river the flow fluvial and in the lower part the flow become torrential. The type of flow may be characterized by the Froude number: when the Froude number is less than 1, i.e. when the fluid velocity is less than the celerity of a surface wave, the flow is termed subcritical or fluvial. Under these conditions, small waves may propagate upstream and downstream. When the Froude number is greater than 1, i.e. when the fluid velocity exceeds the celerity of the surface wave,

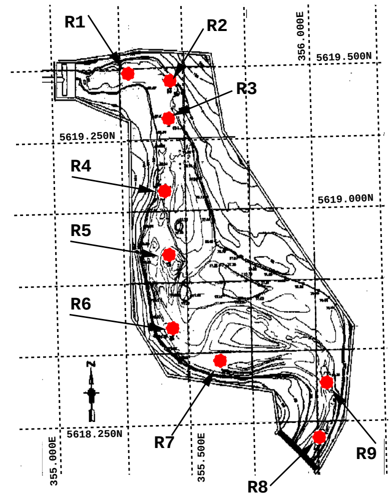

Figure 5: The topography map of the $2km$ Sainte-Marguerite river section. This map has been used to produce the network displayed in Fig. 2 and to adjust the bathymetry and in particular on the river banks. An incorrect value there would lead to a wrong discharge map.

the flow is described as being supercritical or torrential. Under these conditions, an upstream disturbance will propagate rapidly downstream and may cause shocks and instabilities, but the downstream conditions will not have any perceptible effect on the upstream flow behavior (e.g. Guyon *et al.* 1991).

In Fig. 6 are displayed the results of our steady-state simulation. The water level confirms that the errors between the calculation (Fig. 6(a)) and the observations (Fig. 5) at points $R1-9$ are less than the percent. Being given the difficulties of the problem, this is a remarkable performance. The water depth is shown in Fig 6(b) and the water velocity norm in (Fig. 6(c)). The depth is high in upstream where the velocity is also high. The velocity reaches a minimum around the island and in general in the North side whereas it is of higher intensity in the South where the curvature is higher. Thus the simulation correctly describes a meandering flow. The flow downstream is of higher velocity because the rivers becomes narrower there. The energy line is only important upstream (Fig 6(d)) and this is related to the high velocity and high depth there. It justifies *a-posteriori* that we imposed the discharge upstream instead of the water level. As expected, the regime is torrential upstream (see the Froude Number on Fig. 6(e)) but also in the mean part of the river where the water depth is important. Finally a measure of the manning is given by the slope

[2] "Relation niveau-débit" curve supplied by Hydro-Québec. "SM3 - Conditions naturelles Relation Niveau-Débit - Point R9"

Table 1: Table 1: Mean errors between observed (Fig. 5) and calculated (Fig. 6) water level H (m). R9 is the only point for which we know the exact bathymetry.

Point	H_{obs}	H_{cal}	*Error*
R4	76.92	76.50	.42
R5	77.57	78.39	.82
R6	77.33	78.20	.87
R7	77.16	78.16	1.
R8	76.33	77.20	.87
R9	74.30	74.25	.05

of energy grade line ((Fig. 6(f)) and is also corresponding very well to the other quantities.

5 Discussion and Open Questions

In this work, we have used *aquadyn* a Galerkin Finite Element method coupled with a Newton-Raphson time solver to model the Saint-Venant equations on triangular grids. The goal was to simulate a steady state flood propagation over a river with varying width, narrow parts, both torrential and fluvial regimes, complex bathymetry and island. This requires a satisfactory stability scheme during wetting and drying phenomena and ability of handling steep topographical gradients. In the torrential part of the river, where the advection of momentum is dominant, dispersion errors appears under the form of oscillations. Refining the mesh there would certainly eliminate those artefacts but the advection would no more be dominant over a single element. Therefore, an upwinding technique (Hirsh 1988) or an SUPG (Hugues and Brooks 1982) with a fully conservative scheme would be preferable.

The example of a $2km$ section of the Ste-Marguerite River in Canada could be used as a *benchmark* to test new numerical solvers dedicated to flow conditions.

Usually people impose the water level as an entry condition upstream but we found that such a strategy would lead to unstable discharge over the whole domain and also it was necessary to use a wrong water levels both upstream and downstream if one wants to get the correct discharge downstream. However, in this case, not only the water level would be wrong but also the position of the hydraulic jump and therefore the whole physics is wrong. We found that it is rather necessary to impose the discharge upstream and one would get the correct water levels with errors within a few percents.

In sum, despite the fact that the Galerkin scheme is not choc-capturing and thus not naturally adapted to the simulation of torrential regimes, it is of interest that our results can be used as a true benchmark for any standard Galerkin codes.

The next step could be to validate the calibration. This is under way. Another weak point of the procedure that could be improved is the Manning and turbulent coefficients. The efficiency of optimal control techniques for this problem is still an open question (e.g. Poeter and Hill 1997, Louis 1989).

6 Acknowledgements

We would like to thanks Éric McNeil from GeniVar and Michel Leclerc from Hydro-Québec for numerous advices and discussions. The data and curves from Figs. (4), (5) have been provided by Hydro-Québec. This study has been supported by SAJE ("Service d'aide aux Jeunes Entreprises"). Many thanks to Mr Patrice Starace for his help.

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7 Annexe 1: Terminology

- Gravity wave: The free surface of a liquid in equilibrum in a gravitational field is a plane. If, under the action of some external perturbation, the surface is moved from its equilibrum position at some point, motion will occur in the liquid. This motion will be propagated over the whole surface in the form of waves, which are called gravity waves, since they are due to the action of the gravitational field. Gravity waves appear mainly on the surface of the liquid; they affect the interior also, but less and less at greater and greater depths. If the wavelength λ is greater than the water depth h ($\lambda > h$), these waves are called celerity waves. These waves appear in open channel flows and their velocity is given by:

$$v = \sqrt{gh}$$

- Density: The density ρ of a fluid or solid is the mass that it possesses per unit volume.

- Viscosity: Viscosity is the property of a fluid to resists relative motion and deformation and to cause internal shear. Therefore, viscosity is a property exhibited only under dynamic conditions. Acording to Newton, the shear τ at a point within a fluid is proportional to the velocity gradient $\frac{\partial V}{\partial x}$ at that point. The proportionality constant is the dynamic viscosity (μ in $kg - sec/m^2$. When divided by the density, it is the kinematic viscosity ν in m^2/sec. Under ordinary conditions of pressure, viscosity varies only with temperature. The viscosity of a liquid decreases with increasing temperature, the reverse is true for gases.

- Discharge: The discharge Q is the volume of a fluid or solid passing a cross section of a stream per unit time.

- Cross-Sectional Area: The cross-sectional area A is the area of a cross section of the flow normal to the direction of flow.

- Wetted Perimeter: The wetted perimeter P is the length of wetted cross section normal to the direction of flow.

- Hydraulic Radius: The hydraulic radius R is the ratio of the cross-sectional area to wetted perimeter, $R = A/P$.

- Water depth: The depth of flow y is the vertical distance from the bed of a stream to the water surface.

- Energy line: The energy line E' is defined as the total head with respect to the reference level:

$$E' = H + \frac{(U^2 + V^2)}{2g}$$

where: H is the water level and $\frac{(U^2+V^2)}{2g}$ is the velocity head.

- Slope of the Energy Grade Line: The energy grade line is a graphical representation with respect to a selected datum, of the total head or energy possessed by the fluid. For open channel, the energy gradient is located a distance $U^2/2g$ above the free water surface. The slope of the energy grade line is designated by the symbol S_f. The slope of energy line is calculated using the Manning formula:

$$S = n^2 \frac{(U^2 + V^2)^{1/2}}{h^{4/3}}$$

where: n is the Manning coefficient and h is the water depth.

- Froude number: The Froude number is:

$$Fr = \frac{U}{\sqrt{gL}}$$

where g is the gravitational acceleration, U is the mean velocity and L is a characteristic length. The Froude number relates the inertia forces to the gravitational effects and is important wherever the gravity effect is dominating, such as with water waves, flow in open channels, sedimentation in lakes and reservoirs, salt-water intrusions, and the mixing of air masses of specific weights. For open channel flow, $L = h$.

- Reynolds number: The Reynolds number is:

$$Re = \frac{\rho U L}{\mu} = \frac{UL}{\nu}$$

where: U is the velocity, L is the length and ν is the kinematic viscosity. The Reynolds number relates the inertia forces to the viscous forces. In river cases, the flow is fully turbulent and the Reynolds is of the order of one million at least. In this case, one uses a turbulent viscosity and a turbulent Reynolds.

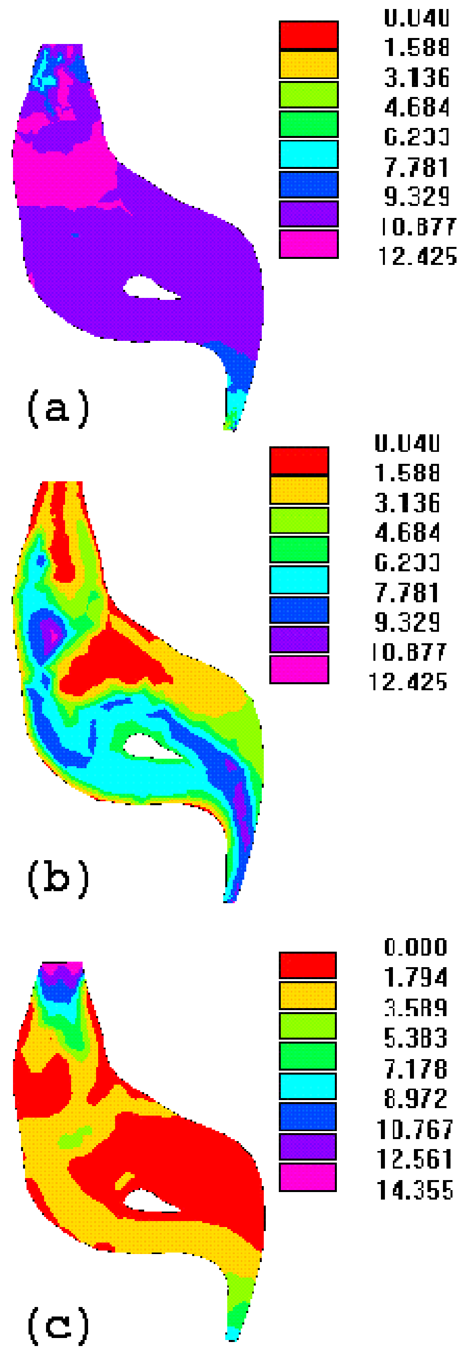

Figure 6: The output from aquadyn after all parameters have been set. (a) Water level (m), (b) Water Depth (m), (c) Water Velocity (m/s)

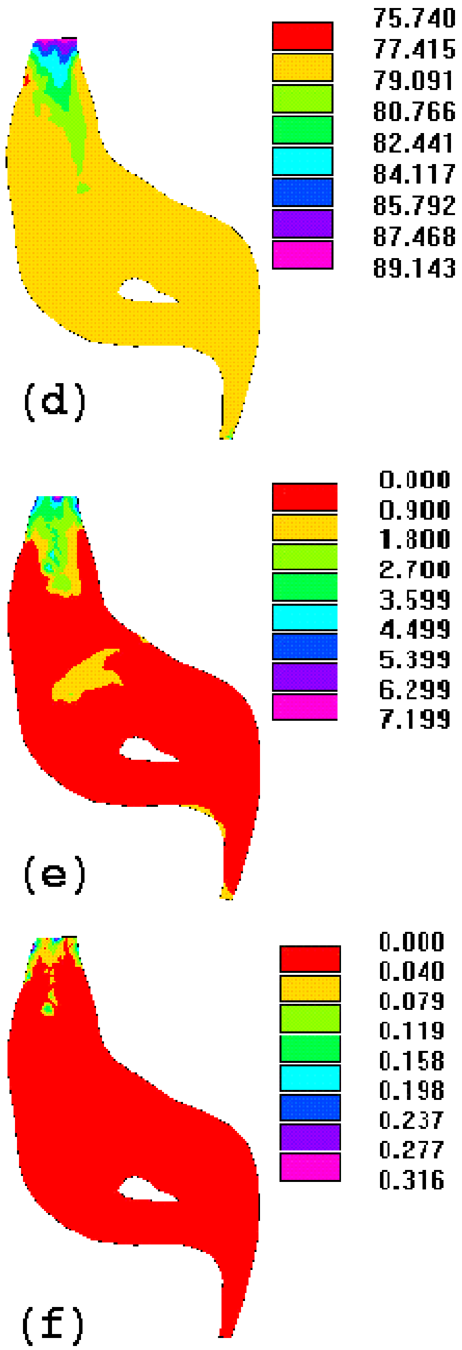

Figure 7: (d) Energy Line (m), (e) Froude Number,
(f) Energy Grade Line.